

This article examines convective mass transfer of an impurity in a shallow bath of molten metal with allowance for the motion of the fusion front during the laser alloying of metals.

The laser alloying of metals is a promising technology. Various schemes are employed to introduce the alloying elements: from previously applied coatings, from the gas phase, by the injection of powder into the laser heating zone, etc. [1]. The redistribution of elements in the molten bath is accomplished to a significant extent by the convective mechanism of mass transfer.

To determine the optimum regimes for the laser alloying of metals, it is necessary to know the character of mixing of the melt and the nonsteady rates of its motion. Among the hydrodynamic processes taking place in the liquid phase during the fusion of metals by laser radiation, one of the most important is thermocapillary convection, i.e., motion of the liquid due to the dependence of the surface tension of the melt on temperature and the nonuniformity of the heating of the free surface. Several studies [2-9] have developed approaches to studying thermocapillary flows, a characteristic feature of which is the constancy (over time) of the surfaces bounding the regions of steady [2, 4-8] and unsteady [3, 9] motion of the liquid. In laser alloying, the character of mixing of the melt and the mass transport of the alloying additions in it are significantly influenced by the movement of the phase transformation boundary into the solid. This movement causes a change over time in the form and path of the particles of the alloying component, which in turn leads to a certain distribution of the components in the remelting zone.

The goal of the present study is to examine the possibilities of deep transport of an alloying component in a melt from the surface by means of thermocapillary mixing during the surface alloying of metals.

FORMULATION AND SOLUTION OF THE PROBLEM

We will examine the nonsteady melting of a massive metal body acted upon by laser radiation. We will assume that the free surface of the melt is planar and that the molten bath is shallow. In a cylindrical coordinate system r, z, φ (with the z axis directed along the depth of the solid phase), the phase boundary is determined by the equation $z = S(r, t)$. We will also assume that the thermal conductivity and diffusivity of the liquid and solid phases and the density, absolute viscosity, and kinematic viscosity of the melt are constant. The energy flux absorbed on the surface $z = 0$ is distributed in accordance with Gauss' law $q(r) = q_0 \exp(-kr^2)$. Thus, the problem is characterized by cylindrical symmetry.

At the moment of the beginning of melting $t = t_m$, the projections of the velocity of the melt v_r, v_z are equal to zero. It is known from boundary-layer theory [10] that the inertial terms in the Navier-Stokes equations can be ignored relative to the viscous terms if the corrected Reynolds number $R^* = (v_r r_f / \nu) (S/r_f)^2 \ll 1$, i.e., if the flow velocities are sufficiently low. Here, $r_f = 1/\sqrt{k}$ is the radius of the hot spot. In this case, the inequality $Re^* Pr \ll 1$ (meaning that conduction predominates over convection) is also satisfied for liquid metals, since molten metals are generally characterized by a low Prandtl number $Pr = \nu/a_1 \ll 1$ [2]. Since the bath is shallow, then $S \ll R \sim r_f$; $\partial^2 v_j / \partial r^2 \sim (1/r) \partial v_j / \partial r \sim v_j / r_f^2 \ll \partial^2 v_j / \partial z^2 \sim v_j / S^2$; $\partial^2 T_i / \partial r^2 \sim (1/r) \partial T_i / \partial r \sim \Delta T_i / r_f^2 \ll \partial^2 T_i / \partial z^2 \sim \Delta T_i / S^2$; $\partial S / \partial r \ll 1$, where $R(t)$ is the radius of the bath; $j = r, z$; $i = 1, 2$. Thus, at $t \geq t_m$, the following mathematical formulation of the problem is valid:

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$$\frac{\partial v_r}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \frac{\partial^2 v_r}{\partial z^2}, \quad (1)$$

$$\frac{\partial v_z}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \frac{\partial^2 v_z}{\partial z^2}, \quad (2)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (rv_r) + \frac{\partial v_z}{\partial z} = 0, \quad (3)$$

$$\left. \frac{\partial v_r}{\partial z} \right|_{z=0} = f(r, t) = \frac{\alpha}{\eta} \left. \frac{\partial T_1}{\partial r} \right|_{z=0}, \quad (4)$$

$$v_r|_{z=S} = v_z|_{z=S} = v_z|_{z=0} = 0, \quad (5)$$

$$\frac{1}{a_1} \frac{\partial T_1}{\partial t} = \frac{\partial^2 T_1}{\partial z^2}, \quad \frac{1}{a_2} \frac{\partial T_2}{\partial t} = \frac{\partial^2 T_2}{\partial z^2}, \quad (6)$$

$$T_1(r, S, t) = T_2(r, S, t) = T_m, \quad (7)$$

$$-\lambda_1 \left. \frac{\partial T_1}{\partial z} \right|_{z=S} = -\lambda_2 \left. \frac{\partial T_2}{\partial z} \right|_{z=S} + \rho L \frac{\partial S(r, t)}{\partial t}, \quad (8)$$

$$-\lambda_1 \left. \frac{\partial T_1}{\partial z} \right|_{z=0} = q(r), \quad (9)$$

$$T_2(r, \infty, t) = T_2(r, z, 0) = T_0. \quad (10)$$

Here, $\alpha = -d\sigma/dT$. It follows from (3) that $v_z/v_r \sim S/r_f$, so that in a shallow bath $v_z \ll v_r$, $\partial v_z/\partial r \ll \partial v_r/\partial z$. Differentiating Eq. (1) with respect to z and Eq. (2) with respect to r , subtracting the second from the first, and making the substitution of variables $x = zq/q_0$, $\tau = t(q/q_0)^2$, we reduce problem (1-10) to the following unidimensional (in the variable x) nonsteady problem:

$$\frac{\partial G}{\partial \tau} = \nu \frac{\partial^2 G}{\partial x^2}, \quad (11)$$

$$G(0, \tau) = g(\tau), \quad (12)$$

$$\int_0^{s(\tau)} dx \int_{s(\tau)}^x G(x_1, \tau) dx_1 = 0, \quad (13)$$

$$\frac{1}{a_1} \frac{\partial u_1}{\partial \tau} = \frac{\partial^2 u_1}{\partial x^2}, \quad \frac{1}{a_2} \frac{\partial u_2}{\partial \tau} = \frac{\partial^2 u_2}{\partial x^2}, \quad (14)$$

$$u_1(s, \tau) = u_2(s, \tau) = T_m, \quad (15)$$

$$-\lambda_1 \left. \frac{\partial u_1}{\partial x} \right|_{x=s} = -\lambda_2 \left. \frac{\partial u_2}{\partial x} \right|_{x=s} + \rho L \frac{ds}{d\tau}, \quad (16)$$

$$-\lambda_1 \left. \frac{\partial u_1}{\partial x} \right|_{x=0} = q_0, \quad (17)$$

$$u_2(\infty, \tau) = u_2(x, 0) = T_0. \quad (18)$$

Here, $G(x, \tau) = (\partial v_r/\partial z - \partial v_z/\partial r)/\beta r \approx (\partial v_r/\partial z)/\beta r$, $\beta = -2ka/\eta$, $g(\tau) = \tau u_1(0, \tau)/\partial \tau$, $s(\tau) = S(\tau, t) \exp(-kr^2)$, $u_1(x, \tau) = T_1(r, z, t)$. An approximate analytical solution of problem (14-18) was obtained in [11]. We change over from G to the functions $v_r(r, z, t)$, $v_z(r, z, t)$ by means of (3) using the following formulas:

$$v_r = \beta r \int_S^z G[z_1 \exp(-kr^2), t \exp(-2kr^2)] dz_1, \quad (19)$$

$$v_z = -\frac{1}{r} \frac{\partial}{\partial r} \left[r \int_0^z v_r(r, z_1, t) dz_1 \right].$$

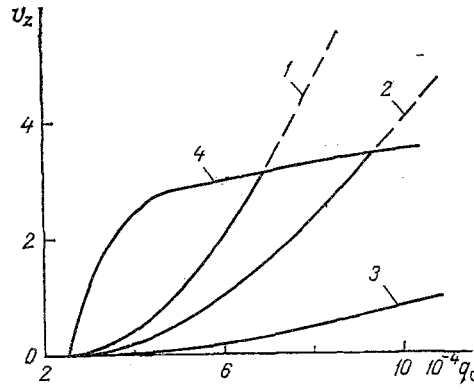


Fig. 1. Projection of velocity v_z at the point r_m, z_m at the moment $t = 1$ msec in the melting of titanium with $k = 200 \text{ cm}^{-2}$ (1), 100 (2), 20 (3); 4) velocity of the fusion front $\partial S/\partial t, v_z, \text{ cm/sec}; q_0, \text{ W/cm}^2$.

Differentiating (13) with respect to τ and using (11) and (12), we obtain the condition

$$\left(1 - \frac{ss}{v}\right) G(s, \tau) - s \left. \frac{\partial G}{\partial x} \right|_{x=s} = g(\tau). \quad (20)$$

An analysis shows that the function $G(x, \tau)$ decreases monotonically for x . It is convex downward and has one zero in the interval $0 \leq x \leq s(\tau)$. Meanwhile, at $\tau = t_m$, i.e., at the moment that melting begins, G depends linearly on $\xi = x/s$. Since melting begins at the point $r = 0$, then $\tau_m = t_m$. Having expanded the functions $g(\tau)$ and $G(x, \tau)$ into series in powers of $\tau - \tau_m$, it can be shown that as the initial condition for unambiguous determination of G , it is sufficient to require that it be finite at $\tau \rightarrow \tau_m$. Thus, to find an approximate solution to problem (11-13), it is convenient to approximate $G(x, \tau)$ in the interval $0 \leq x \leq s(\tau)$ by a second-degree polynomial

$$G(x, \tau) = G_0(\tau) + G_1(\tau)x + G_2(\tau)x^2. \quad (21)$$

We determine the functions G_0, G_1 , and G_2 from (12), (13), and (20). Having inserted (21) into (19), we obtain expressions for the components of the velocity field of the melt:

$$v_r = VW_1, \quad v_z = -UW_2 - SVW_3, \quad (22)$$

where

$$\begin{aligned} V &= \frac{Sf}{6(4+b)}; \quad U = \frac{1}{3(4+b)} \left[\frac{S^2f}{2r} + S \frac{\partial S}{\partial r} f + \frac{1}{2} S^2 \frac{\partial f}{\partial r} - \right. \\ &\quad \left. - \frac{S^2f}{2(4+b)} \frac{\partial b}{\partial r} \right]; \quad W_1 = -6 - b + 6(4+b)y - 9(2+b)y^2 + 4by^3; \\ W_2 &= -6 - b + 3(4+b)y - 3(2+b)y^2 + by^3; \\ W_3 &= y \left[\frac{6+b}{S} \frac{\partial S}{\partial r} - \frac{\partial b}{\partial r} + y \left(3 \frac{\partial b}{\partial r} - 6 \frac{4+b}{S} \frac{\partial S}{\partial r} \right) - y^2 \left(3 \frac{\partial b}{\partial r} - \right. \right. \\ &\quad \left. \left. - 9 \frac{2+b}{S} \frac{\partial S}{\partial r} \right) + y^3 \left(\frac{\partial b}{\partial r} - 4 \frac{b}{S} \frac{\partial S}{\partial r} \right) \right]; \quad y = z/S(r, t); \quad b(\tau) = ss/v. \end{aligned}$$

Discussion of the Results. The flow of the liquid phase is vortical in character. Meanwhile, the melt moves toward the surface in the region near the hot spot and from the center to the edges of the bath on the free surface $z = 0$ [12, 13], since the thermocapillary force is directed toward a decrease in temperature $\alpha > 0$.

From the surface $z = 0$, the alloying components can reach the bottom of the melt only when the maximum positive value of the projection of v_z^{\max} (reached at the point $r_m \approx 3R(t)/4$, $z_m \approx S(r_m, t)/3$) exceeds the velocity of the fusion front $\partial S/\partial t$ (the dashed line in Fig. 1).

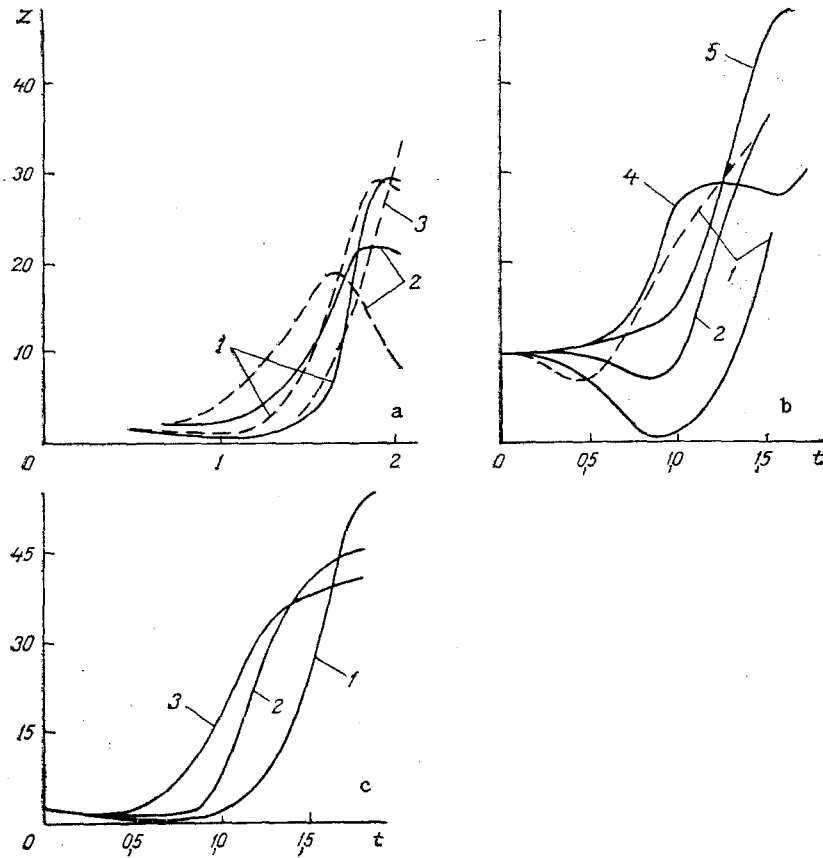


Fig. 2. Time dependence of the coordinate $z(t)$ of a melt particle located at the initial moment $t = 0$ at the point r_0, z_0 in the melting of titanium (1) $r_0/r_F = 0.2$; 2) 0.4; 3) 0.6; 4) 0.8; 5) 1.0): b) $q_0 = 5 \cdot 10^4$ W/cm², $z_0 = 2$ μ m, $k = 150$ cm⁻² (solid lines), 200 cm⁻² (dashed lines); b) $q_0 = 10^5$ W/cm², $z_0 = 10$ μ m, $k = 100$ cm⁻² (solid lines), 150 cm⁻² (dashed line); c) $q_0 = 10^5$ W/cm², $z_0 = 2$ μ m, $k = 100$ cm⁻², z , μ m; t , msec.

Due to the nonsteadiness of melting, the form and size of the vortex (and, thus, the paths of the liquid particles) change over time. This leads to quantitative and qualitative differences between our results and the results obtained in [2-9]. A characteristic feature of the given process is the openness of the particle paths and the increase in their curvature over time. There are also melt particles for which the radius of curvature of the path decreases over time at a certain stage of the process. This occurs in a certain neighborhood of the center of the vortex, which moves in the direction of increasing r and z . Here, as shown by analysis of the velocity field (22), the position of the center of the vortex is described by the approximate relations $r_C \approx R(t)/2$, $z_C \approx S(r_C, t)/3$. Thus, certain paths may turn out to be self-intersecting (curve 4 in Fig. 2b).

Laser alloying of metals is generally done from coatings and coverings, as well as from the gas (liquid) phase or by introducing the alloying component as a powder. If the thickness of the layer of the alloying component on the surface is small compared to the depth of the molten bath, then the averaged thermophysical properties and the viscosity of the melt differ negligibly from the corresponding parameters of the initial material. Otherwise, we should examine thermocapillary convection in a two-layer system. The surface properties of the melt (σ and α , as well as absorptivity) may change appreciably even with small concentrations of the component applied on the surface [2]. Thus, to describe the initial stage of convective mass transfer in the alloying of materials from coverings (coatings), the parameter α for melts of the alloying components should be inserted in the resulting relations. Here, for the sake of definiteness, we will examine laser alloying from the gas phase.

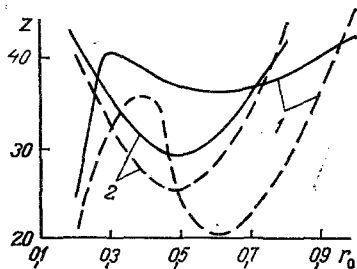


Fig. 3

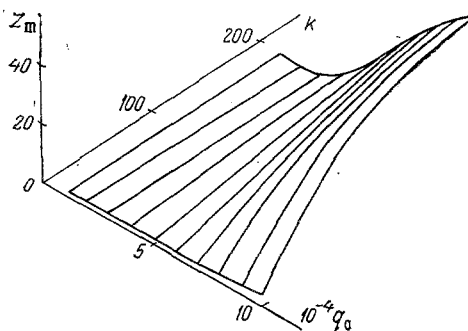


Fig. 4

Fig. 3. The coordinate $z(t)$ of a melt particle at the moment $t = 1.5$ msec in the melting of titanium with $q_0 = 10^5$ W/cm², $z_0 = 2$ μ m (solid lines) and 10 μ m (dashed lines): 1) $k = 100$; 2) 150 cm⁻². r_0 , mm.

Fig. 4. Dependence, on heat flux and the concentration factor, of the maximum depth of penetration of a melt by alloying-component particles, located at the depth $z_0 = 2$ μ m at the moment $t = 0$, at the moment of time $t = 1.5$ msec in the melting of titanium. k , cm⁻²; z_m , μ m.

The alloying component, entering at the free surface $z = 0$, is transferred into the melt by diffusion to the depth $\delta \sim \sqrt{D\Delta t}$, where D is the diffusion coefficient; Δt is the time an elementary volume of the melt is located on the surface, $\Delta t \leq t - t_m$. Meanwhile, it is usually the case that $\delta/S \leq \sqrt{D/a_1} \ll 1$. Liquid particles whose paths at a certain stage approach the surface to within the distance $0 < z < \delta$ transport the alloying component into the bath only in the case when the projection v_z of the velocity of the particles is positive. This occurs far from the center of the hot spot, at $r_0 < r < R(t)$. The quantity $v_z^{\max}(t)$ increases monotonically with time. Meanwhile, at $t \rightarrow t_m$, i.e., at the initial stage of fusion, both v_z^{\max} and the derivative dv_z^{\max}/dt approach zero. Thus, the alloying component is transported to the greatest depth by those liquid particles which during their movement turn out to be in the neighborhood of the point r_m, z_m at the final stage of action of the laser pulse. In Fig. 2, these paths correspond to curves 3 (Fig. 2a), 5, and 1 (Fig. 2b and c). Paths 1 prove to be the most favorable for transporting the alloying component into the melt, as well as for other reasons. In fact, as can be seen from Fig. 2, these particles come closest to the free surface of the liquid phase $z = 0$ during the initial stage of their movement and thus transport the maximum amount of alloying component into the bath.

Figure 2 shows the paths of particles brought into motion after the other particles (curve 3 in Fig. 2a and curve 5 in Fig. 2b) but reaching a significant depth in the melt due to their proximity to the point r_m, z_m at a certain stage. There are also particles which manage to make almost a complete revolution and return to the neighborhood of their initial position during the time of action of the pulse (dashed curve 2 in Fig. 2a).

Thus, along with laser alloying regimes which ensure a relatively uniform distribution of the alloying component in the bath, there are other regimes which produce a nonmonotonic distribution with a minimal concentration in the vicinity of the center of the vortex.

It is evident from Fig. 3 that the depth of penetration of an alloying addition into a bath by means of thermocapillary mixing depends heavily on the initial position of the elementary volumes accomplishing the mass transport. The existence of minima on the given curves means that particles having r_0 as their initial coordinates turn out to be far from the point r_m, z_m in the course of their movement, i.e., they turn out to be in the region least suitable for deep mass transfer.

It is convenient to use Fig. 4 to analyze the feasibility of deep convective transport of alloying additions into a melt by means of thermocapillary convection. Figure 4 shows different regimes of laser irradiation. It is evident that an increase in the parameters q_0 and k intensifies mass transfer, since this is accompanied by an increase in the thermocapillary force (4) which induces the melt to move. This force is proportional to the temperature gradient on the surface $\partial T_1/\partial r \sim q_0 \sqrt{k a_1 (t - t_m)}/\lambda$.

It should be noted that the method described here for calculating the nonsteady melting of metals by changing over to new variables x and τ , with the above assumptions, is valid for an arbitrary law governing the spatial distribution of heat flux (9). Thus, in a number of cases of laser irradiation, the function $q(r)$ is conveniently approximated in the form of a superposition of Gaussian sources. Here, the minimum characteristic linear dimension $q(r)$ should be considerably greater than the depth of the heat-affected zone. Thus, generally speaking, the given method is inapplicable in regard to allowing for the fine-scale structure of laser radiation. Substantial heat-flux gradients over small distances result in high values of the temperature gradient $\partial T/\partial r$, i.e., the heat fluxes in the plane $z = 0$. This tends to smooth the temperature on corresponding small sections of the surface. Thus, in such cases, it is possible to examine the melting of a solid by a heat flow averaged over small-scale gradients.

NOTATION

r, z, φ , cylindrical coordinates; t , time; T_i , temperature of the liquid ($i = 1$) and solid ($i = 2$) phases; $q(r)$, absorbed energy flux; k , concentration factor; T_m , melting point; L , heat of fusion; ρ , density; λ_i, α_i , thermal conductivity and diffusivity; T_0 , initial temperature; η, ν , absolute and kinematic viscosities of the melt; v_r, v_z , projections of the melt velocity on the coordinate axes r and z ; p , pressure; σ , surface tension.

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